Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

2018-19

EXM/P/09/01

Year and Program:2018-19 B.Sc.II	**School of Science**	**Department of Mathematics**
Course Code: MTS 201	**Course Title: Mathematics III**	**Semester – III**
Day and Date: saturday 01/06/2019	**End Semester Examination (ESE)**	**Time:** 2·30 to 3·00 pm
		Max Marks: 100
PRN/Exam seat No:		**Answer booklet No:**
Student's signature:		**Invigilator signature:**

A

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question has four choices out of which only one is the correct
4) Tick mark (√) the correct alternative which should be answered in question paper itself and submit to the Jr.Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicates full marks
7) Use **Blue ball pen** only.

Q.1		**Choose the correct Alternative for following questions.**	Marks	Bloom's Level	CO
	i)	If A,B,C are sets, then I) $A\setminus(B\cup C)=(A\setminus B)\cup(A\setminus C)$ II) $A\setminus(B\cap C)=(A\setminus B)\cap(A\setminus C)$ a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO1
	ii)	I) The set $\mathbb{Z}$ of all integers is not denumerable. II) The set $\mathbb{N}\times\mathbb{N}$ is denumerable. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO1
	iii)	If S is a countable set. I) There exist a surjection of N onto S. II) There exists an injection of S onto N. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO1

ESE

iv)	Suppose that S and T are sets and that $T \subseteq S$, then I) If S is a finite set, then T is a finite set. II) If T is infinite set, then S is an infinite set. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO1
v)	The set A of all real numbers x such that $2x+3 \le 6$ then …… a) $A=\{x \in R : x \le 3/2\}$ b) $A=\{x \in R : x < 3/2\}$ c) $A=\{x \in R : x > 3/2\}$ d) $A=\{x \in R : x \ge 3/2\}$	01	L1	CO2
vi)	Consider the set $A=\left\{\frac{1}{\sqrt{n^2+1}} : n \in N\right\}$. Then A is …. a)Countable b) Uncountable c) Finite d) None of these	01	L1	CO2
vii)	I) If x and y are complex then $\lvert\lvert x\rvert-\lvert y\rvert\rvert=\lvert x-y\rvert$ II) If $x \in R^k$ and $y \in R^k$ then $\lvert x-y\rvert^2+\lvert x+y\rvert^2=2\lvert x\rvert^2+2\lvert y\rvert^2$ a) Both I and II true b) only I true c) Both I and II false d) only II true	01	L1	CO2
viii)	I) If $a,b \in \mathbb{R}$,then $a^2+b^2=0$ if and only if $a=0$ or $b=0$ II) If $a,b \in \mathbb{R}$ and $0 \le a < b$ then $a^2 < ab < b^2$. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO2
ix)	The limit of the sequence $\{x_n\}$ $where\, x_n=\frac{(-1)^n}{2n-1}$ is…… a) 1 b) -1 c) 0 d) does not exist	01	L1	CO3

x)	I) Every bounded sequence in R contains convergent subsequence. II) A sequence $\{P_n\}$ Converges to p, if every subsequence of $\{P_n\}$ converges to p. a) Both I and II true b) only I true c) Both I and II false d) only II true	01	L1	CO3
xi)	I) Every convergent sequence need not be Cauchy. II) Every Cauchy sequence is convergent. a) Both I and II true b) only I true c) Both I and II false d) only II true	01	L1	CO3
xii)	If $S_n = (-1)^n \left[1+\left(\frac{1}{n}\right)\right]$ then $\lim_{n\to\infty} S_n = -----$ a) 1 b) 0 c) -1 d) Does not exist	01	L1	CO3
xiii)	I) The p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}, p>1$ converges. II) The Harmonic series $\sum_{n=1}^{\infty}\frac{1}{n^p}, 0<p\le 1$ diverges. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO4
xiv)	I) The series $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ converges. II) The series $\sum_{n=1}^{\infty}\frac{1}{n!}$ diverges. a) Only I true b)Only II true c)Both I and II true d) Both I and II false	01	L1	CO4
xv)	The series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ converges to …. a) 1 b) 0 c) -1 d) e	01	L2	CO4

xvi)	If $S_n = 1 + \left[\frac{(-1)^n}{n} \right]$ then sequence $\{S_n\}$ is	01	L1	CO4

a) converges to 1,bounded & has infinite range

b) converges to 0, bounded & has infinite range

c) converges to 1, bounded and has finite range

d) converges to 0, bounded and has finite range

xvii	$\lim_{n\to\infty} \left\{ \left(\frac{1}{n} \right) \sin(nx+n) \right\} = ... \ \forall x \in R$	01	L1	CO5

a) 0 b) 1 c) -1 d) Does not exist

xviii	$\lim_{n\to\infty} \frac{nx}{1+n^2x^2} = ...; \textit{for all } x \in \mathbb{R}$	01	L1	CO5

a) 1 b) 0 c) -1 d) e

xix)	Let $a_1 = -2, a_{n+1} = \frac{n\,a_n}{n+1}$ then $\lim_{n\to\infty} a_n = ...$	01	L1	CO5

a) 1 b) 0 c) -1 d) e

xx)	$\lim_{n\to\infty} \frac{3x^2+7}{n} = ..., \forall x \in \mathbb{R}$	01	L1	CO5

a) 1 b)0 c)-1 d) Does not exist

*****

Year and Program:2018-19 B.Sc.II | **School of Science** | **Department of Mathematics**

Course Code: MTS 201 | **Course Title: Mathematics III** | **Semester – III**

Day and Date: Saturday 01/06/2019 | **End Semester Examination (ESE)** | **Time:** 3-00 to 5-30 pm.

Max Marks: 100

Instructions:
1) All questions are compulsory.
2) Assume suitable data wherever necessary.
3) Figures to the right indicate full marks.

B

Q.2		**Solve any Two.**	Marks	Bloom's Level	CO
	a)	Prove that The set Q of all rational numbers is countable.	05	L2	CO1
	b)	Prove that the set $\mathbb{N}\times\mathbb{N}$ is countable, when $\mathbb{N}$ is set of natural numbers.	05	L2	CO1
	c)	Show that for each $n\in N$, the sum of the squares of the first n natural numbers is given by $\frac{1}{6}n(n+1)(2n+1)$	05	L2	CO1
Q3		**Solve any Two.**			
	a)	If A and B are bounded subsets of real numbers, then prove that $A\bigcap B$ and $A\bigcup B$ are also bounded.	05	L2	CO2
	b)	State and prove Archimedean property.	05	L2	CO2
	c)	If x and y are any real numbers with property $x<y$,then there exist a rational number $r\in\mathbb{Q}$ such that $x<r<y$,	05	L2	CO2
Q4		**Solve any Two.**			
	a)	Let (x_n) be a sequence of positive real numbers such that $L=\lim\limits_{n\to\infty}\left(\frac{x_{n+1}}{x_n}\right)$ exists. If $L<1$, then $X=(x_n)$ converges and $\lim\limits_{n\to\infty}(x_n)=0$.	05	L2	CO3
	b)	Show that, A sequence of real numbers is Cauchy if and only if it is convergent sequence.	05	L2	CO3
	c)	Let the sequence $X=(x_n)$ converges to x. Then the sequence $(\lvert x_n\rvert)$ of absolute value converges to $\lvert x\rvert$.	05	L2	CO3

ESE

			Marks	Level	CO
Q5					
	a)	**Solve any Three.**			
	i)	Test the given series for convergence $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$	06	L3	CO4
	ii)	Let $\sum x_n$ be an absolutely convergent series in $\mathbb{R}$. Then show that any rearrangement $\sum y_k$ of $\sum x_n$ converges to same value.	06	L3	CO4
	iii)	Discuss convergence and divergence of p-series by using Raabe's test.	06	L3	CO4
	iv)	Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ converges.	06	L2	CO4
	b)	If a series in $\mathbb{R}$ is absolutely convergent, then prove that it is convergent.	07	L3	CO4
Q.6					
	a)	**Solve any Three.**			
	i)	Let (f_n) be a sequence of bounded functions on $A \subseteq \mathbb{R}$. Then prove that this sequence converges uniformly on A to a bounded function f if and only if for each $\in > 0$ there is number $H(\in)$ in $\mathbb{N}$ such that for all $m,n \geq H(\in)$ then $\|f_m - f_n\|_A \leq \in$.	06	L3	CO5
	ii)	Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that converges (f_n) uniformly on A to a function $f : A \to \mathbb{R}$, Then prove that f is continuous on A.	06	L3	CO5
	iii)	A sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.	06	L3	CO5
	iv)	Define pointwise convergence of sequence of functions. Also show that $\lim_{n\to\infty}\frac{x}{n} = 0$ for $x \in \mathbb{R}$.	06	L2	CO5
	b)	Let (f_n) be a sequence of functions in $\mathfrak{R}[a,b]$ and suppose that (f_n) converges uniformly on $[a,b]$ to f. Then show that $f \in \mathfrak{R}[a,b]$ and $\int_a^b f = \lim_{n\to\infty}\int_a^b f_n$.	07	L4	CO5

ESE