



Sanjay Ghodawat University, Kolhapur

Established as State Private University under Govt. of Maharashtra. Act No XL,
2017

2018-19
EXM/P/09/00

Year and Program:
2018-19 B. Sc. II

School of Science

Department of Mathematics

Course Code – MTS 208

Course Title – Number Theory

Semester – IV

Day and Date – Tuesday
28/05/2019

End Semester Examination

Time: 2.30 to 3.00 pm.

PRN number –

Seat no-

Max Marks: 100

Answer Booklet No.-

Students' Signature -

(A)

Invigilator's Signature –

Instructions:

- 1) All questions are compulsory.
- 2) Attempt Q.1 within first 30 minutes.
- 3) Each MCQ type question is followed by four plausible alternatives, Tick (\checkmark) the correct one.
- 4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
- 5) If you tick more than one option it will not be evaluated.
- 6) Figures to the right indicate full marks.
- 7) Use Blue ball pen only.

| Q.1 | Tick the correct answers. | Marks | Bloom's Level | CO |
|------|---|-------|------------------|-----|
| i) | Which one of the following Diophantine Equations, cannot be solved? a) $6x + 51y = 22$ b) $2x + 3y = 32$ c) $33x + 14y = 115$ d) $4x + 3y = 15$ | 02 | L ₂ | CO1 |
| ii) | If $d = (a, b) = ax + by$ then $(x, y) =$ _____ a) 0 b) 1 c) -1 d) d | 01 | L ₁ | CO1 |
| iii) | The quotient and remainder when -1 is divided by 3 is _____ a) -1 and -1 b) -1 and 2 c) 1 and 2 d) 1 and -2 | 01 | L ₂ | CO1 |

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- iv) The greatest common divisor of $3^{13} \cdot 5^{17}$ and $2^{12} \cdot 3^5$ is _____ 02 L₂ CO1
 a) 3^0 b) 3^1
 c) 3^3 d) 3^5
- v) Which one of the following congruence relations is not possible? 02 L₄ CO2
 a) $6 \equiv 2 \pmod{4}$ b) $5 \equiv 2 \pmod{3}$
 c) $10 \equiv 5 \pmod{5}$ d) $10 \equiv 2 \pmod{3}$
- vi) The number of factors of a prime number are: 01 L₂ CO2
 a) 2
 b) 3
 c) Depends on the prime number
 d) None of the mentioned
- vii) Prime Numbers are _____ 01 L₁ CO2
 a) Finite b) Infinite
 c) 100000 d) 255
- viii) If $ac \equiv bc \pmod{n}$ and $d = \underline{\hspace{2cm}}$ then $a \equiv b \pmod{\frac{n}{d}}$ 02 L₂ CO2
 a) $d = \text{g.c.d.}(c, n)$ b) $d = \text{g.c.d.}(a, n)$
 c) $d = \text{g.c.d.}(b, n)$ d) $d = \text{g.c.d.}(a, b)$
- ix) $\mu(8) = \underline{\hspace{2cm}}$ 02 L₂ CO3
 a) 1 b) 0
 c) -1 d) 2
- x) $\phi(13) = \underline{\hspace{2cm}}$ 02 L₂ CO3
 a) 13 b) 12
 c) 14 d) 1
- xi) The number of positive divisors of 12 is _____ 02 L₂ CO3
 a) 5 b) 20
 c) 6 d) 7
- xii) The sum of positive divisors of 20 is _____ 02 L₂ CO3
 a) 40 b) 25
 c) 20 d) 12

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Number Theory

Semester – IV

Day and Date Tuesday
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Time: 3:00 to 5:30 PM. Max Marks: 100

(B)

Instructions:

- 1) All questions are compulsory.
- 2) Use of non-programmable calculator is allowed.
- 3) Figures to the right indicate full marks.

| Q.2 | Attempt any two of the following | Marks | Bloom's Level | CO |
|------|--|-------|------------------|-----|
| a) | Prove by mathematical induction that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ | 08 | L ₃ | CO1 |
| b) | Calculate g.c.d. of 1479 and 272, also express it as their linear combination. | 08 | L ₄ | CO1 |
| c) | Let S be a set of positive integers with properties a) $1 \in S$ b) $k \in S \Rightarrow k + 1 \in S$ then prove that S is the set of all positive integers. | 08 | L ₃ | CO1 |
| | | | | |
| Q.3 | Attempt any two of the following | Marks | Bloom's Level | CO |
| a) | Show that congruence relation is an equivalence relation. | 08 | L ₃ | CO2 |
| b) | Solve $18x \equiv 30 \pmod{42}$ | 08 | L ₄ | CO2 |
| c) | If $a \equiv b \pmod{n}$ then prove that $\text{g.c.d.}(a, n) = \text{g.c.d.}(b, n)$. | 08 | L ₂ | CO2 |
| | | | | |
| Q.4 | A Attempt any one of the following | Marks | Bloom's Level | CO |
| a) | Define relatively prime integers. If $\text{g.c.d.}(a, b) = d$ then show that | 08 | L ₂ | CO1 |
| | $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$ | | | |

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b) For any integer x show that $(a, b) = (a, b + ax)$ 08 L₃ CO1

B Attempt any one of the following

a) Solve 08 L₄ CO2

$$x \equiv 3 \pmod{5}$$

$$x \equiv 6 \pmod{7}$$

b) Let a, b and c denote integers and let n be a positive integer. Then show that. 08 L₁ CO2

i) If $a \equiv b \pmod{n}$, then $a + c \equiv b + c \pmod{n}$.

ii) If $a \equiv b \pmod{n}$, then $ac \equiv bc \pmod{n}$.

Q.5 Attempt any two of the following

a) Find the remainder when 10^{241} is divided by 7. 08 L₄ CO3

b) Calculate a) $\tau(180)$ b) $\sigma(180)$ 08 L₂ CO3

c) Prove that σ and τ are multiplicative. 08 L₃ CO3

Q.6 Attempt any two of the following

a) If $n = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \dots \dots p_r^{k_r}$ is the prime factorization of $n > 1$ then 08 L₃ CO3

prove that

$$a) \tau(n) = (k_1 + 1) \cdot (k_2 + 1) \dots \cdot (k_r + 1)$$

$$b) \sigma(n) = \left(\frac{p_1^{k_1+1} - 1}{p_1 - 1} \right) \cdot \left(\frac{p_2^{k_2+1} - 1}{p_2 - 1} \right) \dots \cdot \left(\frac{p_r^{k_r+1} - 1}{p_r - 1} \right)$$

b) Prove that μ is multiplicative function. 08 L₃ CO3

c) Calculate a) $\phi(5040)$ b) $\phi(3000)$ 08 L₅ CO3

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